Grade 4 Math Unit 4-Number and Operations - Fractions

UNIT OVERVIEW

In Grade 4, instructional time should focus on three critical areas. This unit will address Critical Focus Area #2, **Developing an understanding of fractions equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers.** (See Connections for explanation)

This unit addresses work in the following clusters:

- Extend understanding of fraction equivalence and ordering
- Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
- Understand decimal notation for fractions, and compare decimal fractions.

STANDARDS

CC_Common Core State Standards - Mathematics (2010) - Grade 4

Domain 4.NF Number and Operations-Fractions

Cluster Statement: ***Extend understanding of fraction equivalence and ordering.***

Standard 4.NF.1 Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

Standard 4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Cluster Statement: ***Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.***

Standard 4.NF.3 Understand a fraction a/b with a > 1 as a sum of fractions 1/b.

4.NF.3.a Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

4.NF.3.b Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.

4.NF.3.c Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

4.NF.3.d Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

Standard 4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

4.NF.4.a Understand a fraction a/b as a multiple of 1/b.

4.NF.4.b Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number.

4.NF.4.c Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.

Cluster Statement: ***Understand decimal notation for fractions, and compare decimal fractions.***

Standard 4.NF.5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.

Standard 4.NF.6 Use decimal notation for fractions with denominators 10 or 100.

Standard 4.NF.7 Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.

CONTENT ELABORATIONS

4.NF.1	In this unit students extend their work on **4.OA.4**. **4.NF.1** This standard refers to visual fraction models. this includes area models, linear models (number lines) or it could be a collection/sets models. This work extends the work in Grade 3 by using additional denominators (5, 10, 12, and 100). **MP.2, MP.4, MP.7, MP.8** should be emphasized.
4.NF.2	**4.NF.2** calls for students to compare fractions by creating visual fraction models or finding common denominators or numerators. Students' experience should focus on visual fraction models rather than algorithms. When tested, models may or may not be included. Students should learn to draw fraction models to help them compare and use reasoning skills based on fraction benchmarks. Students must also recognize that they must consider the size of the whole when comparing fractions (ie, 1/2 and 1/8 of two medium pizzas is very different from 1/2 of one medium and 1/8 of one large). Record the results of comparisons with symbols >, =, or < and justify the conclusions, e.g., by using visual fraction model. **MP.2, MP.4, MP.5, MP.7** should be emphasized.
4.NF.3	**4.NF.3** A fraction with a numerator of one is called a unit fraction. When students investigate fractions other than unit fractions, such as 2/3, they should be able to decompose the non-unit fraction into a combination of several unit fractions. (2/3 = 1/3 + 1/3) Being able to visualize this decomposition into unit fractions helps students when adding or subtracting fractions. Students need multiple opportunities to work with mixed numbers and be able to decompose them in more than one way. Students may use visual models to help develop this understanding. Mixed numbers are introduced for the first time in fourth grade. Students should have ample experiences of adding and subtracting mixed numbers where they work with numbers or convert mied numbers into improper fractions. Keep in mind Concrete-Representation-Abstract (CRA) approach to teaching fractions. Students need to be able to 'show' their thinking using concrete and/or representations before they move to abstract thinking. A separate algorithm for mixed numbers in addition and subtraction is not necessary. Students will tend to add or subtract the whole numbers first and then work with the fractions using the same strategies they have applied to problems that contained only fractions. **MP.1, MP.2, MP.4, MP.5, MP.6, MP.7, MP.8** should be emphasized.
4.NF.4a-c	**4.NF.4** Students need many opportunities to work with problems in context to understand the connections between models and corresponding equations. Contexts involving a whole number times a fraction lend themselves to modeling and examining patterns. **4.NF.4a** This standard builds on students' work of adding fractions and extending that work into multiplication. **4.NF.4b** This standard extends the idea of multiplication as repeated addition. For example, 3 x (2/5) = 2/5 + 2/5 + 2/5 = 6 x (1/5). Students are expected to use and create visual fraction models to multiply a whole number by a fraction. **4.NF.4c** This standard calls for students to use visual fraction models (Area, Linear and Set Models) to solve word problems related to multiplying a whole number by a fraction. **MP.1, MP.2, MP.4, MP.5, MP.6, MP.7, MP.8** should be emphasized.
4.NF.5	**4.NF.5** This standard continues work of equivalent fractions by having students change fractions with a 10 in the denominator into equivalent fraction that have a 100 in the denominator. In order to prepare for work with decimals, experiences that allow students to shade decimal grids (10x10 grids) can support this work. Student experiences should focus on working with grids rather than algorithms. Students can also use base ten blocks and other place value models to explore the relationship between fractions with denominators of 10 and denominators of 100. Base Ten Blocks: students may represent 3/10 with 3 longs and may also write the fraction as 30/100 with the whole in this case being the flat (the flat represents one hundred units with each unit equal to one hundredth). Students begin to make connections to the place value chart. Students make connections between fractions with denominators of 10 and 100 and the place value chart. By reading fraction names, students say 32/100 as thirty-two hundredths and rewrite this as 0.32 or represent it on the place value chart model. Students use the representations explored in this standard to understand 32/100 can be expanded to 3/10 and 2/100. Students represent values such as 0.32 or 32/100 on a number line. 32/100 is more than 30/100 (or 3/10) and less than 40/100 (or 4/10). It is closer to 30/100 so it would be placed on the number line near that value. This work in fourth grade lays the foundation for performing operations with decimal numbers in fifth grade. **MP.2, MP.4, MP.5, MP.7** should be emphasized.

4.NF.6	**4.NF.6** Students make connections between fractions with denominators of 10 and 100 and the place value chart. By reading fractions names. By reading frations names, students say 32/100 as thirty-two hundreths and rewrite this as 0.32 or represent it on a place value model. Students use the representations explored in 4.NF.5 to understand 32/100 can be expanded to 3/10 and 2/100. Students represent values such as 0.32 or 32/100 on a number line. 32/100 is more than 30/100 (or 3/10) and less than 40/100 (or 4/10). It is closer to 30/100 so it would be placed on the number line near that value. **MP.2, MP.4, MP.5, MP.7** should be emphasized.
4.NF.7	**4.NF.7** Students build area and other models to compare decimals. Through these experiences and their work with fraction models, they build the understanding that comparisons between decimals or fractions are only valid when the whole is the same for both cases. When the wholes are the same, the decimals or fractions can be compared. Ex. Draw a model to show that 0.3 < 0.5. (Students would sketch two models of approximately the same size to show the area that represents three-tenths is smaller than the area that represents five-tenths.) **MP.2, MP.4, MP.5, MP.7** should be emphasized.

UNIT VOCABULARY

factor pairs	simplest form	improper fraction
prime number	greatest common factor	like fractions
composite number	least common multiple	decimal
numerator	benchmark fractions	tenth
denominator	mixed number	hundredth
equivalent fractions		

BIG IDEAS

ENDURING UNDERSTANDINGS	ESSENTIALS QUESTIONS Choose a few questions based on the needs of your students

●Mixed numbers and improper fractions can be used interchangeably.
●Fractions can be represented visually and in written form.
●Fractions with differing parts can be the same size.
●Fractions of the same whole can be compared.
●Fractions can be represented visually and in written form.
●Fractional amounts can be added and/or subtracted.
●Mixed numbers can be added and/or subtracted.
●Mixed numbers and improper fractions can be used interchangeably.
●Mixed numbers can be ordered by considering the whole number and the fraction.
●Fractional numbers and mixed numbers can be added and/or subtracted.
●Fractions, like whole numbers can be unit intervals on a number line
●Fractional amounts can be added and/or multiplied.
●Fractions can be expressed as decimals.
●Decimals can be represented visually and in written form.
●Decimals are a part of the base ten system.
●Tenths can be expressed using an equivalent fraction with a denominator of 100.
●Comparisons of two decimals are only valid when the two decimals refer to the same whole.

●How can different fractions name the same amount?
● How can I use operations to model real-world fractions?
●How are fractions and decimals related?

CONNECTIONS

In **Critical Focus Area #2**, students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal and they develop methods for generating and recognizing equivalent fractions. Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.

The work in this unit's clusters are connected to:
3.NF.3, 4.NF1-2, 4.MD.4, 5NBT.5-7

Standards for Mathematical Practice (SMP)
MP.1 Make sense of problems and persevere in solving them
MP.2 Reason abstractly and quantitatively
MP.3 Construct viable arguments and critique the reasoning of others
MP.4 Model with mathematics
MP.5 Use appropriate tools strategically
MP.6 Attend to precision
MP.7 Look for and make use of structure (Deductive reasoning)
MP.8 Look for and express regularity in repeated reasoning (Inductive Reasoning)

EXTEND UNDERSTANDING OF FRACTION EQUIVALENCE AND ORDERING		
	CONTENT	**SKILLS**
4.NF.4	Recognize and generate equivalent fractions	Recognize and generate equivalent fractions 1. Recognize and identify equivalent fractions with unlike denominators 2. Explain why a/b is equal to (nxa)/(nxb) by using fraction models with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. (Ex: Use fraction strips to show why ½=2/4=3/6=4/8) 3. Use visual fraction models to show why fractions are equivalent (ex: ¾ = 6/8) 4. Generate equivalent fractions using visual fraction models and explain why they can be called "equivalent".
4.NF.2	Compare two fractions with different numerators and different denominators	Compare two fractions with different numerators and different denominators 1. Recognize fractions as being greater than, less than, or equal to other fractions. 2. Use benchmark fractions such as ½ for comparison purposes. 3. Make comparisons based on parts of the same whole. 4. Compare two fractions with different numerators, e.g. by comparing to a benchmark fraction such as ½. 5. Compare two fractions with different denominators, e.g. by creating common denominators, or by comparing to a benchmark fraction such as ½. 6. Justify the results of a comparison of two fractions, e.g. by using a visual fraction model.
BUILD FRACTIONS FROM UNIT FRACTIONS BY APPLYING AND EXTENDING PREVIOUS UNDERSTANDINGS OF OPERATIONS ON WHOLE NUMBERS		
	CONTENT	**SKILLS**
4.NF.3a	Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.	Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. 1. Understand that accumulating unit fractions (1/b) results in a fraction (a/b), where a is greater than 1. 2. Students extend previous understandings about how fractions are built from unit fractions, composing (joining) fractions from unit fractions, and decomposing (separating) fractions into unit fractions 3. Using fraction models, reason that addition of fractions is joining parts that are referring to the same whole. 4. Using fraction models, reason that subtraction of fractions is separating parts that are referring to the same whole.

4.NF.3b	Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation	Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation 1. Add and subtract fractions with like denominators. 2. Recognize multiple representations of one whole using fractions with the same denominator. 3. Using visual fraction models, decompose a fraction into the sum of fractions with the same denominator in more than one way. 4. Record decompositions of fractions as an equation and explain the equation using visual fraction models.
4.NF.3c	Add and subtract mixed numbers with like denominators	Add and subtract mixed numbers with like denominators 1. Add and subtract mixed numbers with like denominators by using properties of operations and the relationship between addition and subtraction. 2. Replace mixed numbers with equivalent fractions, using visual fraction models. 3. Replace improper fractions with a mixed number, using visual fraction models. 4. Add and subtract mixed numbers by replacing each mixed number with an equivalent fraction.
4.NF.3d	Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators.	Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators. 1. Add and subtract fractions with like denominators. 2. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, by using visual fraction models and equations to represent the problem.
4.NF.4a	Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product 5 x ¼, recording the conclusion by equation 5/4 = 5 x (1/4)	Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product 5 x ¼, recording the conclusion by equation 5/4 = 5 x (1/4) 1. Represent a fraction a/b as a multiple of 1/b (unit fractions). For example, represent 5/4 as an accumulation of five ¼'s. 2. Extend previous understandings about how fractions are built from unit fractions, using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number. 3. Apply multiplication of whole numbers to multiplication of a fraction by a whole number using visual fraction models. (For example, just as students know that four 3's can be represented by 4x3, students know that five 1/4's is 5 x 1/4 which is 5/4.)
4.NF.4b	Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number.	Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. 1. Explain that a multiple of a/b is a multiple of 1/b (unit fraction) using a visual fraction model. 2. Multiply a fraction by a whole number by using the idea that a/b is a multiple of 1/b. For example, use a visual fraction model to express 3 x (2/5) as 6 x (1/5) recognizing this product as (6/5).
4.NF.4c	Solve word problems involving multiplication of a fraction by a whole number.	Solve word problems involving multiplication of a fraction by a whole number. 1. Multiply a fraction by a whole number. 2. Use fraction models and equations to represent the problem. 3. Solve word problems involving multiplication of a fraction by a whole number.
UNDERSTAND DECIMAL NOTATION FOR FRACTIONS, AND COMPARE DECIMAL FRACTIONS		
CONTENT		**SKILLS**

4.NF.5	Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.	Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. 1. Rename and recognize a fraction with a denominator of 10 as a fraction with a denominator of 100. 2. Recognize that two fractions with unlike denominators can be equivalent. 3. Use knowledge of renaming tenths to hundredths to add two fractions with denominators 10 and 100.
4.NF.6	Use decimal notation for fractions with denominators 10 or 100.	Use decimal notation for fractions with denominators 10 or 100. 1. Explain the values of digits in the decimal places. 2. Read and write decimals through hundredths. 3. Rename fractions with 10 and 100 in the denominator as decimals. 4. Recognize multiple representations of fractions with denominators 10 or 100. 5. Represent fractions with denominators 10 or 100 with multiple representations and decimal notation. 6. Explain how decimals and fractions relate.
4.NF.7	Compare two decimals to hundredths by reasoning about their size.	Compare two decimals to hundredths by reasoning about their size. 1. Recognize that comparisons are valid only when the two decimals refer to the same whole. 2. Compare two decimals to hundredths by reasoning about their size. 3. Record the results of comparisons with the symbols >, =, or <. 4. Justify the conclusions using visual models and other methods.

UNIT RESOURCES

Common Core Model Curriculum
McGraw-Hill, **My Math** Chapters 8-10
Georgia Math frameworks, Grade 4 Units 3-7
Number Talks by Sherry Parrish
fraction tile/bars
circular fraction models
number lines
Smart Board manipulatives
Discovery Math
http://illuminations.nctm.org/Activity.aspx?id=3510